SOME MEASURES OF CONSENSUS GENERATED BY DISTANCES ON WEAK ORDERS

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Abstract

In this paper we introduce some classes of consensus measures based on metrics on weak orders, and we analyze some of their properties. Taking into account these consensus measures, we have also introduced indices of contribution to consensus for each decision maker for prioritizing them in order of their contributions to consensus. We have also proposed a decision procedure where the individual opinions are weighted by the corresponding indices of contribution to consensus.

Keywords: Consensus, Metrics, Weak orders.

1 Introduction

When the members of a committee show their preferences over a set of alternatives, it is important to find not only a collective preference but to measure the agreement or disagreement between the members of the committee and the result (see Nurmi [21]). In this paper we focus our attention in how to measure consensus in groups of voters when they show their preferences over a fixed set of alternatives or candidates by means of weak orders (complete preorders).

We note that consensus has different meanings 1. One of them is related to iterative procedures where voters must change their preferences to improve agreement. Usually, a moderator advise voters to modify some opinions (see, for instance, Eklund, Rusinowska and de Swart [10]). However, in this paper consensus is related to the degree of agreement in a committee, and voters do not need to change their preferences.

1 For a survey and references, see Martínez-Panero [18].

From a technical point of view, it is interesting to note that the problem of measuring the concordance or discordance between two linear orders has been widely explored in the literature. In this way, different rank correlation indices have been considered for assigning grades of agreement between two rankings (see Kendall and Gibbons [17]). Some of the most important indices in this context are Spearman’s rho [24], Kendall’s tau [16], and Gini’s cograduation index [13]. On the other hand, some natural extensions of the above mentioned indices have been considered for measuring the concordance or discordance among more than two linear orders (see Hays [14]). For details and references, see for instance Borroni and Zenga [6].

In the field of Social Choice, Bosch [7] introduced the notion of consensus measure as a mapping that assigns a number between 0 and 1 to every profile of linear orders, satisfying three properties: (strong) unanimity (in every subgroup of voters, the highest degree of consensus is only reached whenever all individuals have the same ranking), anonymity (the degree of consensus is not affected by any permutation of voters) and neutrality (the degree of consensus is not affected by any permutation of alternatives).

Recently, Alcalde and Vorsatz [1] have introduced some consensus measures in the context of linear orders –related to some of the above mentioned rank correlation indices– and they provide some axiomatic characterizations.

In this paper we extend the Bosch’s notion of consensus measure to the context of weak orders (indifference among different alternatives is allowed) 2, and we consider some additional properties that such measures could fulfill. After that, we introduce a class of consensus measures based on distances among individual and collective scores generated by an aggregation operator.

2 Recently, García-Lapresta [11] has introduced a class of agreement measures in the context of weak orders when voters classify alternatives within a finite scale defined by linguistic categories with associated scores. These measures are based on distances among individual and collective scores generated by an aggregation operator.
sus measures based on the distances among individual weak orders\(^3\). We pay special attention to some specific metrics based on discrete, Minkowski, Chebishev and cosine distances.

We also introduce two consensus measures related to the Borda rule \(^5\). Although the Borda rule \(^4\) was initially introduced under the assumption of individual preferences are linear orders, there exist some extensions to weak orders (see Black \[^4\], Gärdenfors \[^12\] and Young \[^25\], among others). We follow one extended Borda count where relative positions of tied alternatives are the average of corresponding positions after a linearization of the initial weak order (see Smith \[^23\], Black \[^4\] and Cook and Seiford \[^9\]).

The mentioned consensus measures related to the Borda rule are defined through distances among individual voters by their relative contributions to consensus. Moreover, we also propose a decision procedure that weights the opinions of weak orders on \(X\). Given a profile \(R = (R_1, \ldots, R_m)\) of weak orders, where \(R_i\) contains the preferences of the voter \(v_i\). Given a profile \(R = (R_1, \ldots, R_m)\), we denote \(R^{-1} = (R_1^{-1}, \ldots, R_m^{-1})\).

If \(\pi\) is a permutation on \(\{1, \ldots, m\}\) and \(\emptyset \neq I \subseteq V\), we denote \(R_{\pi} = (R_{\pi(1)}, \ldots, R_{\pi(m)})\) and \(I_{\pi} = \{v_{\pi^{-1}(i)} \mid v_i \in I\}\), i.e., \(v_j \in I_{\pi} \iff v_{\pi(j)} \in I\).

Given a permutation \(\sigma\) on \(\{1, \ldots, n\}\), we denote with \(R_{\sigma} = (R_{\sigma}^1, \ldots, R_{\sigma}^n)\) the profile that results of recalling in \(R\) the alternatives according to \(\sigma\), i.e., \(x_i R_k x_j \iff x_{\sigma(i)} R_{\sigma}^k x_{\sigma(j)}\) for all \(i, j \in \{1, \ldots, n\}\) and \(k \in \{1, \ldots, m\}\).

The cardinal of any subset \(I\) is denoted by \(|I|\). With \(\mathcal{P}(V)\) we denote the power set of \(V\), i.e., \(I \in \mathcal{P}(V) \iff I \subseteq V\), and \(\mathcal{P}_2(V) = \{I \in \mathcal{P}(V) \mid |I| \geq 2\}\). Notice that \(|\mathcal{P}_2(V)| = |\mathcal{P}(V)| - |V| - 1 = 2^m - m - 1\).

### 2.1 Codification of weak orders

We now introduce a system for codifying linear and weak orders by means of vectors which represent the relative position of each alternative in the corresponding order. Similar procedures have been considered in the generalization of scoring rules from linear orders to weak orders (see Smith \[^23\], Black \[^4\] and Cook and Seiford \[^9\], among others).

Given a profile \((R_1, \ldots, R_m) \in L(X)^m\) of linear orders, consider the mapping \(o_l : X \rightarrow \{1, \ldots, n\}\) which assigns the position of each alternative in \(R_i\). Thus, the vector \((o_l(x_1), \ldots, o_l(x_n)) \in \{1, \ldots, n\}^n\) determines the corresponding linear order, and we denote \(R_i \equiv (o_l(x_1), \ldots, o_l(x_n))\).

There does not exist a unique system for codifying weak orders. We propose one based on linearizing the weak order and to assign each alternative the average of the positions of the alternatives within the same equivalence class. Thus, given a profile of weak orders \((R_1, \ldots, R_m) \in W(X)^m\), the mapping

\[
o_l : X \rightarrow \{1, 1.5, 2, 2.5, \ldots, n - 0.5, n\}
\]

assigns the relative position of each alternative in \(R_i\).

So, if 7 alternatives are arranged in the weak order \(x_2 \sim x_3 \sim x_5 > x_1 > x_4 \sim x_7 > x_6\), then this weak order is codified by the vector \((4, 2, 2, 5.5, 2, 7, 5.5)\).

**Remark 1** Because of ties, weak orders on \(W(X)\) not necessarily assign all positions \(1, \ldots, n\) to alternatives. However, it is clear that the sum of all positions is \(1 + 2 + \cdots + n\), as in linear orders. Consequently, for...
Remark 2 Notice that if \( R_i \in W(X) \) is codified by the vector \( (a_0(x_1), \ldots, a_0(x_n)) \), then \( R_i^{-1} \) is codified by \( (n + 1 - a_0(x_1), \ldots, n + 1 - a_0(x_n)) \).

2.2 Metrics

We now introduce a simple procedure for constructing a metric on \( W(X) \) from a metric on \( \mathbb{R}^n \).

Definition 1 Given a metric \( d : \mathbb{R}^n \times \mathbb{R}^n \rightarrow [0, \infty) \), the metric on \( W(X) \) induced by \( d \) is the mapping \( d : W(X) \times W(X) \rightarrow [0, \infty) \) defined by 
\[
d(R_1, R_2) = d((o_1(x_1), \ldots, o_1(x_n)), (o_2(x_1), \ldots, o_2(x_n))) \quad \text{for all} \quad R_1, R_2 \in W(X)
\]

Example 1 Typical examples of metrics in \( \mathbb{R}^n \) are the following:

1. The discrete metric \( d' \),
\[
d'((a_1, \ldots, a_n), (b_1, \ldots, b_n)) = \begin{cases} 
1, & \text{if } (a_1, \ldots, a_n) \neq (b_1, \ldots, b_n), \\
0, & \text{if } (a_1, \ldots, a_n) = (b_1, \ldots, b_n).
\end{cases}
\]

2. For every \( p \geq 1 \), the Minkowski metric \( d_p \),
\[
d_p((a_1, \ldots, a_n), (b_1, \ldots, b_n)) = \left( \sum_{i=1}^{n} |a_i - b_i|^p \right)^{\frac{1}{p}}.
\]

For \( p = 1 \) and \( p = 2 \) we have the Manhattan and Euclidean metrics, respectively.

3. The Chebyshev metric \( d_\infty \),
\[
d_\infty((a_1, \ldots, a_n), (b_1, \ldots, b_n)) = \max \{ |a_1 - b_1|, \ldots, |a_n - b_n| \}.
\]

4. The cosine metric \( d_c \),
\[
d_c((a_1, \ldots, a_n), (b_1, \ldots, b_n)) = \frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\sum_{i=1}^{n} a_i^2} \sqrt{\sum_{i=1}^{n} b_i^2}}.
\]

Definition 2 A metric \( d : \mathbb{R}^n \times \mathbb{R}^n \rightarrow [0, \infty) \) is neutral if for every permutation \( \sigma \) on \{1, \ldots, n\}, it holds
\[
d((a_{\sigma(1)}, \ldots, a_{\sigma(n)}), (b_{\sigma(1)}, \ldots, b_{\sigma(n)})) = d((a_1, \ldots, a_n), (b_1, \ldots, b_n)),
\]
for all \( (a_1, \ldots, a_n), (b_1, \ldots, b_n) \in \mathbb{R}^n \).

Remark 3 All the metrics introduced in Example 1 are neutral.

3 Consensus measures

Consensus measures have been analyzed by Bosch [7] in the context of linear orders. We now extend this concept to the framework of weak orders. Notice that we use a unanimity condition weaker than that of Bosch.

Definition 3 A consensus measure on \( W(X)^m \) is a mapping \( M : W(X)^m \times \mathcal{P}_2(V) \rightarrow [0, 1] \) that satisfies the following conditions:

1. Weak unanimity. For every \( R \in W(X)^m \), it holds
\[
M(R, V) = 1 \Leftrightarrow R_1 = \cdots = R_m.
\]

2. Anonymity. For all permutation \( \pi \) on \{1, \ldots, m\}, \( R \in W(X)^m \) and \( I \in \mathcal{P}_2(V) \), it holds
\[
M(R_{\pi}, I) = M(R, I).
\]

3. Neutrality. For all permutation \( \sigma \) on \{1, \ldots, n\}, \( R \in W(X)^m \) and \( I \in \mathcal{P}_2(V) \), it holds
\[
M(R^{\sigma}, I) = M(R, I).
\]

Weak unanimity means that the maximum consensus in the set of all decision makers is only achieved when all opinions are the same. Anonymity requires symmetry with respect to decision makers, and neutrality means symmetry with respect to alternatives.

We now introduce other properties that a consensus measure could satisfy.

Definition 4 Let \( M : W(X)^m \times \mathcal{P}_2(V) \rightarrow [0, 1] \) be a consensus measure.

1. \( M \) satisfies strong unanimity if for all \( R \in W(X)^m \) and \( I \in \mathcal{P}_2(V) \), it holds
\[
M(R, I) = 1 \Leftrightarrow R_i = R_j \quad \text{for all} \quad v_i, v_j \in I.
\]

2. \( M \) satisfies maximum dissension if for all \( R \in W(X)^m \) and \( v_i, v_j \in V \) such that \( i \neq j \), it holds \( M(R, \{v_i, v_j\}) = 0 \Leftrightarrow (R_i, R_j) \in L(X) \) and \( R_j = R_i^{-1} \).

3. \( M \) is reciprocal if for all \( R \in W(X)^m \) and \( I \in \mathcal{P}_2(V) \), it holds
\[
M(R^{-1}, I) = M(R, I).
\]
4. $\mathcal{M}$ is homogeneous if for all $R \in W(X)^m$, $I \in \mathcal{P}_2(V)$ and $t \in \mathbb{N}$, it holds

$$\mathcal{M}^t(R, t I) = \mathcal{M}(R, I),$$

where $\mathcal{M}^t : W(X)^m \times \mathcal{P}_2(t V) \rightarrow [0, 1]$, $tR = (R, \ldots, R) \in W(X)^m$ is the profile defined by $t$ copies of $R$ and $t I = I \cup \ldots \cup I$ is the set of voters defined by $t$ copies of $I$.

Strong unanimity means that the maximum consensus in every subset of decision makers is only achieved when all opinions are the same. Obviously, strong unanimity implies weak unanimity.

Maximum dissension means that in each subset of two voters\(^5\), the minimum consensus is only reached whenever preferences of voters are linear orders and each one is the inverse of the other.

Reciprocity means that if all individual weak orders are reversed, then the consensus does not change.

Homogeneity means that if we replicate a subset of voters, then the consensus in that group does not change.

### 3.1 Consensus measures based on metrics

**Definition 5** Given a metric $\bar{d}$ on $W(X)$, the mapping

$$\mathcal{M}_{\bar{d}} : W(X)^m \times \mathcal{P}_2(V) \rightarrow [0, 1]$$

is defined by

$$\mathcal{M}_{\bar{d}}(R, I) = 1 - \frac{\sum_{i \neq j, i < j} \bar{d}(R_i, R_j)}{\binom{|I|}{2}} \cdot \Delta_n,$$

where

$$\Delta_n = \max\{d(R_i, R_j) \mid R_i, R_j \in W(X)\}.$$

**Proposition 1** For every metric $\bar{d}$ on $W(X)$, $\mathcal{M}_{\bar{d}}$ satisfies strong unanimity and anonymity.

If $\mathcal{M}_{\bar{d}}$ is neutral, then we say that $\mathcal{M}_{\bar{d}}$ is the consensus measure associated with $\bar{d}$.

**Proposition 2** If $\bar{d} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow [0, \infty)$ is a neutral metric, then $\mathcal{M}_{\bar{d}}$ is a consensus measure.

**Proposition 3** If $\bar{d}$ is the metric on $W(X)$ induced by $d^*$, $d_p$, with $p \geq 1$, $d_\infty$ or $d_0$, then $\mathcal{M}_{\bar{d}}$ is a reciprocal consensus measure.

\(^5\)It is clear that a society reach maximum consensus when all the opinions are the same. However, in a society with more than two members it is not an obvious issue to determine when there is minimum consensus (maximum disagreement).

**Proposition 4** If $\bar{d}$ is the metric induced by $d_p$, with $p > 1$, or $d_c$, then $\mathcal{M}_{\bar{d}}$ satisfies the maximum dissenion property.

**Proposition 5** If $\bar{d}$ is the metric induced by $d^*, d_1$ or $d_\infty$, then $\mathcal{M}_{\bar{d}}$ does not satisfy the maximum dissenion property.

**Proposition 6** The consensus measure $\mathcal{M}_{\bar{d}}$ is not homogeneous for any distance $\bar{d}$ on $W(X)$.

**Proposition 7** If $R \in L(X)^2$ and $R_2 = R_1^{-1}$, then for any distance $\bar{d}$ on $W(X)$ it holds:

$$\lim_{t \to \infty} \mathcal{M}_{\bar{d}}(t R, t I) = \frac{1}{2}.$$

### 3.2 Borda consensus measures

Given a profile of weak orders $(R_1, \ldots, R_m) \in W(X)^m$ and $I \in \mathcal{P}_2(V)$, let $\sigma_i^* : X \rightarrow \mathbb{R}$ be the mapping defined by

$$\sigma_i^*(x_j) = \frac{1}{|I|} \sum_{i \in I} a_i(x_j),$$

which assigns the average position to each alternative for the set of voters $I$.

Notice that the weak order $R_i^*$ defined by $x_j, R_i^* x_k \Leftrightarrow \sigma_i^*(x_j) \leq \sigma_i^*(x_k)$ corresponds to the Borda rule for the set of voters $I$.

**Definition 6** The absolute Borda consensus measure is the mapping

$$\mathcal{M}_B : W(X)^m \times \mathcal{P}_2(V) \rightarrow [0, 1]$$

defined by

$$\mathcal{M}_B(R, I) = 1 - \frac{\sum_{i \in I} d(R_i, R_i^*)}{|I| \cdot \Delta_n},$$

where $R_i^*$ is the weak order associated with the outcome provided by the Borda rule for the whole set of voters $V$, $\bar{d}$ the metric on $W(X)$ induced by the Euclidean metric and

$$\Delta_n = \max\{d(R_i, R_j) \mid R_i, R_j \in W(X)\}.$$

**Proposition 8** $\mathcal{M}_B$ is a consensus measure that satisfies reciprocity and homogeneity.

**Proposition 9** $\mathcal{M}_B$ does not satisfy strong unanimity.

**Definition 7** The relative Borda consensus measure is the mapping

$$\mathcal{M}_B' : W(X)^m \times \mathcal{P}_2(V) \rightarrow [0, 1]$$

where

$$\mathcal{M}_B'(R, I) = 1 - \frac{\sum_{i \in I} d(R_i, R_i^*)}{|I| \cdot \Delta_n}.$$
defined by
\[ M'_B(R, I) = 1 - \frac{\sum_{i \in I} d(R_i, R'_i)}{|I| \cdot \Delta_n}, \]
where \( R'_i \) is the weak order associated with the outcome provided by the Borda rule for the set of voters \( I \) and \( d \) the metric on \( W(X) \) induced by the Euclidean metric.

**Proposition 10** \( M'_B \) is a consensus measure that satisfies strong unanimity, reciprocity and homogeneity.

**Proposition 11** \( M_B \) and \( M'_B \) do not satisfy the maximum dissension property.

We summarize the properties of the analyzed consensus measures in Table 1.

<table>
<thead>
<tr>
<th>( M ) for ( d' )</th>
<th>S.U.</th>
<th>M.D.</th>
<th>R.</th>
<th>H.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_d ) for ( d'_c )</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( M_d ) for ( d_1 )</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( M_d ) for ( d_p, p &gt; 1 )</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( M_d ) for ( d_{\infty} )</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( M_d ) for ( d_c )</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( M_B )</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>( M'_B )</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

S.U.: Strong unanimity; M.D.: Maximum dissension; R.: Reciprocity; H.: Homogeneity.

## 4 Favouring consensus in group decision making

We now introduce an index for each voter that shows what is his/her contribution to consensus. This index has some similarities with the Shapley value in cooperative game theory.

**Definition 8** Given a consensus measure \( M \), the mapping \( c : W(X)^m \rightarrow [-1, 1]^m \) is defined by \( c(R) = (c_1(R), \ldots, c_m(R)) \) with
\[
c_i(R) = \frac{\sum_{I \in S(i)} \left( M(R, I \cup \{v_i\}) - M(R, I) \right)}{|S(i)|},
\]
i = 1, \ldots, m, where \( S(i) = \{I \in P_d(V) | v_i \notin I\}\).

We say that \( c_i(R) \) is the marginal contribution to consensus of voter \( v_i \) with respect to \( R \in W(X)^m \).

**Proposition 12** Let \( d : W(X) \times W(X) \rightarrow [0, \infty) \) be a metric. For the consensus measure \( M_d \) and \( M_B \), it holds \( c_1(R) + \cdots + c_m(R) = 0 \), for every profile \( R \in W(X)^m \).

We can use the vector \( c(R) \) for prioritizing the voters in order of their contribution to consensus, as suggested by Cook, Kress and Seiford [8]. In this way, we introduce a new index
\[
c'_i(R) = c_i(R) - \min\{c_1(R), \ldots, c_m(R)\},
\]
for every \( i \in \{1, \ldots, m\} \). We now define a weight for each voter:
\[
w_i(R) = \begin{cases} 
c'_i(R) + \cdots + c'_m(R) & \text{if } c'_1(R) + \cdots + c'_m(R) \neq 0 \\
\frac{1}{m}, & \text{if } c'_1(R) + \cdots + c'_m(R) = 0,
\end{cases}
\]
for \( i = 1, \ldots, m \).

Notice that \( w_i(R) \in [0, 1] \) for every \( i \in \{1, \ldots, m\} \), and \( w_1(R) + \cdots + w_m(R) = 1 \).

Now, for each decision maker \( v_i \in V \) we multiply the position of each alternative \( o_i(x_j) \) by his/her weight \( w_i(R) \) for assigning collective positions to the alternatives:
\[
O(x_j) = \sum_{i=1}^{m} w_i(R) \cdot o_i(x_j), \quad j = 1, \ldots, n.
\]

Thus, we can order the alternatives through the weak order \( \succeq \) on \( X \) defined by
\[
x_j \succeq x_k \Leftrightarrow O(x_j) \leq O(x_k).
\]

### 4.1 An illustrative example

To illustrate the above decision making procedure, we consider the set of voters \( V = \{v_1, v_2, v_3, v_4, v_5\} \) that rank order the alternatives of \( X = \{x_1, \ldots, x_7\} \) by means of the weak orders included in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Individual orders</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
</tr>
<tr>
<td>( x_2 ) ( x_3 ) ( x_5 )</td>
</tr>
<tr>
<td>( x_1 )</td>
</tr>
<tr>
<td>( x_4 ) ( x_7 )</td>
</tr>
<tr>
<td>( x_6 ) ( x_4 ) ( x_6 ) ( x_7 )</td>
</tr>
<tr>
<td>( x_6 ) ( x_7 )</td>
</tr>
</tbody>
</table>

If we consider the consensus measure \( M_d \) for \( d_c \), we obtain the marginal contributions to consensus included in Table 3.
The marginal contributions to consensus are given by:
\[ c_1(R) = 0.05114639 \]
\[ c_2(R) = 0.03468305 \]
\[ c_3(R) = 0.01296238 \]
\[ c_4(R) = 0.07872037 \]
\[ c_5(R) = -0.17751219 \]

These marginal contributions to consensus induce the weights included in Table 4.

We now weigh the position of each alternative by these weights included in Table 4.

Taking into account these aggregated positions we obtain the following linear order on \( X \):
\[ M_d \text{ for } d_c : x_3 \succ x_5 \succ x_1 \succ x_2 \succ x_4 \succ x_7 \succ x_6 \]

If we apply the same procedure taking into account consensus measures \( M_d \) for metrics \( d_p \), with \( p = 2 \), and \( d_\infty \), and the Borda consensus measures, then we obtain the following orders:
\[ M_d \text{ for } d_2 : x_5 \succ x_3 \succ x_1 \succ x_2 \succ x_4 \succ x_7 \succ x_6 \]
\[ M_d \text{ for } d_\infty : x_5 \succ x_2 \succ x_1 \sim x_3 \succ x_4 \succ x_7 \succ x_6 \]
\[ M_B : x_3 \succ x_5 \succ x_1 \succ x_2 \succ x_4 \succ x_7 \succ x_6 \]
\[ M'_B : x_5 \succ x_3 \succ x_1 \succ x_2 \succ x_4 \succ x_7 \succ x_6 \]

Remark 4 The Borda count is equivalent to use
\[ O_B(x_j) = \frac{1}{m} \sum_{i=1}^{m} o_i(x_j), \quad j = 1, \ldots, n, \]
and the weak order \( \succeq_B \) on \( X \) defined by
\[ x_j \succeq_B x_k \iff O_B(x_j) \leq O_B(x_k). \]

If we apply the Borda count to the weak orders of Table 2, then we would obtain the following collective order:
\[ \succeq_B : x_1 \succ x_3 \succ x_2 \succ x_5 \succ x_4 \succ x_7 \succ x_6 \]

quite different to those obtained by applying the proposed weighted decision making procedure.

4.2 Some remarks

It is interesting to emphasize the following features of our decision making procedure:

- Due to the marginal contribution to consensus indices and corresponding weights are usually irrational numbers, so when the number of decision makers is high, it is unlikely that the decision procedure provides ties among alternatives.

- Since the proposed decision procedure penalizes individuals who are far from majority positions, this provides incentives for decision makers to moderate their opinions which, otherwise, may be excluded or underestimated.

- It is easy to implement computer programs for generating the final ranking on the set of alternatives.

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